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Formation of a proto-Jovian envelope for various planetary accretion rates

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Abstract. The formation of a proto-Jovian envelope has been simulated on the basis of a core accretion model and the maximum mass that a proto-Jovian planet can have while keeping its envelope gravitationally stable, called the critical core mass, has also been investigated extensively over a wide range of the core accretion rate. The value of the critical core mass has been found to depend strongly on the core accretion rate; for example, it is less than or equal to 1 $M_\oplus$ for the typical accretion rates for Uranus and Neptune. Furthermore, through simulations of the quasi-static evolution of the envelope beyond the critical core mass, we have found that the characteristic times of envelope contraction are 6×10^5 years, 7×10^6 years and 5×10^7 years for the cases where the core accretion rates are 1×10^{-6} $M_\oplus$ per year, 1×10^{-7} $M_\oplus$ per year and 1×10^{-8} $M_\oplus$ per year, respectively. Also, in the last case, the core mass of the Jovian planet can be estimated to be about 4 $M_\oplus$. We conclude that if a given one of the Jovian planets of our solar system has a core smaller than about 5 $M_\oplus$, it is very hard to see how the core could have attracted a gaseous envelope from our solar nebula and formed the Jovian envelope. Determination of the sizes of the cores in our Jovian planets should give fruitful information for the theory of the formation of our solar system.

In the standard theory of the formation of our solar system, the ‘Kyoto model’, the formation of the giant planets (Jupiter, Saturn, Uranus and Neptune) is studied on the basis of a core accretion model. At first, the proto-Jovian planets, which are also called proto-Jovian cores, were formed through the coalescence of planetesimals and, when their masses M_{core} became as large as about 10 $M_\oplus$, rapid gas accretion began to form their massive envelopes from the solar nebular gas [1, 2]. The core mass at the onset of the rapid gas accretion is called the *critical core mass*. This model, which we will call the *standard model* hereafter, was widely accepted for the two reasons that it seemed plausible for explaining (i) the estimated M_{core}-values of these planets (10 to 30 $M_\oplus$) and (ii) their large solid/gas ratios (compared with the solar abundance ratio) [3]. However, recent models of the interior structure of Jupiter and Saturn [4, 5] have not given such large M_{core}-values; they have given estimates of the values of M_{core} of 3 to 10 $M_\oplus$ for Jupiter and 1 to 13 $M_\oplus$ for Saturn. If each giant planet has a core smaller than 10 $M_\oplus$, the standard model cannot explain why such a small core is capable of obtaining its large amount of the gaseous envelope, because it should never become larger than the critical core mass. Furthermore, the serious dilemma that the formation of a 10 $M_\oplus$ core requires 5×10^7 to 1×10^{10} years [6]—much longer than the lifetime of the nebular gas around a young solar-type star ($\sim10^5$ to 10^7 years) [7]—makes some theorists doubt the validity of the standard model itself.

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However, is the value of the critical core mass ($M_{\rm core}^{\rm crit}$) equal to or larger than 10 $M_\oplus$ in *any* case? In the context of the standard model, planetesimals falling onto the core play a primary role in determining the structure of the envelope and its stability. They transform their kinetic energy into the thermal energy at the bottom of the envelope. The released thermal energy warms up the envelope and stabilizes it against the gravity force of the core. Thus the core accretion rate ($\dot{M}_{\rm core}$) which determines the amount of thermal energy released at the bottom of the envelope per unit time must have a great influence on the resultant critical core mass. Although the importance of $\dot{M}_{\rm core}$ has already been recognized in [2], $M_{\rm core}^{\rm crit}$ has been examined only for small ranges of $\dot{M}_{\rm core}$, such as from 1×10^{-5} to 1×10^{-7} $M_\oplus$ per year. Since $\dot{M}_{\rm core}$ given by the Kyoto model varies from $\sim 10^{-6}$ $M_\oplus$ per year for Jupiter to $\sim 10^{-10}$ $M_\oplus$ per year for Neptune, this work is insufficient for determining the actual value of $M_{\rm core}^{\rm crit}$.

Recently, Pollack *et al* [8] have simulated the formation of the giant planets taking into consideration three combined processes, namely the gas accretion of a core, the enhancement of the cross section of the core for capturing planetesimals owing to the drag effect of the envelope and the decline of the surface density of the planetesimals due to the capture by the core. The latter two effects change $\dot{M}_{\rm core}$ considerably. Also, the structure of the envelope and the total mass of the proto-planet play important roles in the latter two processes. The views of Pollack *et al* on the three processes are important and unique. However, their work involves a big problem. As they mentioned, we have no clear and established view of the planetesimal accumulation process in the later stages of the accumulation. So they assumed that the core should capture all of the planetesimals existing in the feeding zone, which is determined only by the criterion of the energy. However, it is known that, because of the drag effect of the nebular gas on the planetesimals and gravitational effects from other planets, the proto-planet cannot capture all of the planetesimals in the feeding zone [6, 9]. Therefore there is still uncertainty as regards $\dot{M}_{\rm core}$—and the gas accretion rate too, because the gas accretion rate strongly depends on $\dot{M}_{\rm core}$. In addition, Pollack *et al* considered that formation of a core of mass greater than 10 $M_\oplus$ is essential to explain our Jovian planets, and concluded that surface densities two to four times as large as that of the minimum-mass solar nebula are necessary. But, as mentioned before, the recent estimation does not indicate such a large core mass. So we have to confirm whether a mass distribution of planetesimals a few times larger than the minimum-mass solar nebula is a necessary condition. Also, we do not know how large it is possible for $\dot{M}_{\rm core}$ to be. Therefore, it is important to investigate the gas accretion process over a wide range of values of $\dot{M}_{\rm core}$ in a simple model, rather than using complete but complicated models with a specific and complicated $\dot{M}_{\rm core}$. The purpose of this work is to determine $M_{\rm core}^{\rm crit}$ over a wide range of $\dot{M}_{\rm core}$ and make it clear whether a proto-Jovian core could capture the known envelope from our solar nebula during the nebular lifetime.

A critical core mass is found for one value of $\dot{M}_{\rm core}$ as follows. Assuming that the envelope is in a hydrostatic equilibrium state and that the luminosity, which is the amount of energy passing through a sphere in the envelope per unit time, is spatially constant. Under these assumptions, the equation of mass conservation, the hydrostatic equilibrium, the energy transfer and the equation of state for the ideal gas are obtained. The critical core mass is found as the maximum mass of the core for which a static solution for the structure of the envelope can be obtained. On the other hand, the evolution of the gaseous envelope around the core after the value of $M_{\rm core}$ reaches $M_{\rm core}^{\rm crit}$ is sought using another numerical method. The evolution is quasi-static and the luminosity in the envelope is supplied by the gravitational contraction of the envelope as well as planetesimals falling onto the core. Thus, the equation of energy conservation is needed instead of the assumption that the luminosity is spatially constant in the envelope. This set of partial differential equations is solved numerically in the so-called relaxation method [11] commonly used to study stellar evolution.

Table 1. The relation between the critical core mass $M_{\rm core}^{\rm crit}$ and the core accretion rate $\dot{M}_{\rm core}$. In this table, $M_{\rm env}^{\rm crit}$ and $L^{\rm crit}$ are the corresponding envelope mass and the luminosity, respectively.

$\dot{M}_{\rm core}$ ($M_\oplus$ per year)	1×10^{-6}	1×10^{-7}	1×10^{-8}	1×10^{-9}	1×10^{-10}
$M_{\rm core}^{\rm crit}$ ($M_\oplus$)	12.1	7.43	3.87	1.83	0.818
$M_{\rm env}^{\rm crit}$ ($M_\oplus$)	7.22	4.62	2.24	0.917	0.349
$L^{\rm crit}$ (erg s^{-1})	7.51×10^{26}	5.32×10^{25}	3.37×10^{24}	2.03×10^{23}	1.20×10^{22}

The critical core masses for five values of $\dot{M}_{\rm core}$ are listed in table 1. The critical core mass decreases as $\dot{M}_{\rm core}$ decreases. Roughly speaking, $M_{\rm core}^{\rm crit}$ is proportional to $(\dot{M}_{\rm core})^{0.3}$. Here we must emphasize that $M_{\rm core}^{\rm crit}$ is not always 10 $M_\oplus$. In particular, its value is equal to or less than 1 $M_\oplus$ in the cases where $\dot{M}_{\rm core}$ is less than 1×10^{-9} $M_\oplus$ per year, which is the typical value of $\dot{M}_{\rm core}$ in the later stages of formation of Uranus and Neptune. Hereafter, we denote the envelope mass for $M_{\rm core} = M_{\rm core}^{\rm crit}$ as $M_{\rm env}^{\rm crit}$. As shown in table 1, the $M_{\rm env}^{\rm crit}/M_{\rm core}^{\rm crit}$ ratios are about 50% for all of the values of $\dot{M}_{\rm core}$. Thus, $M_{\rm env}^{\rm crit}$ is only a few Mars masses in the case where $M_{\rm core}^{\rm crit} < 1$ $M_\oplus$. Therefore we have to investigate how long it takes for the core to attract a large amount of the envelope.

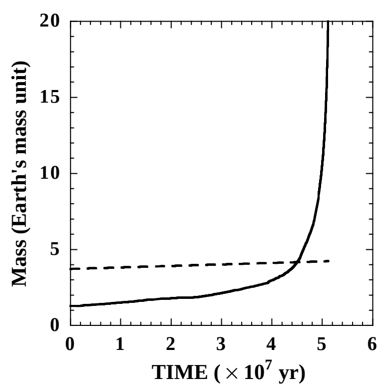

Figure 1. The evolution of the envelope mass (solid line) and that of the core mass (dashed line) in the case where $\dot{M}_{\rm core} = 1 \times 10^{-8}$ $M_\oplus$ per year. They are plotted as functions of time.

Table 2. The growth times τ_g for three values of $\dot{M}_{\rm core}$ at the time when $M_{\rm core} = M_{\rm core}^{\rm crit}$.

$\dot{M}_{\rm core}$ ($M_\oplus$ per year)	1×10^{-6}	1×10^{-7}	1×10^{-8}
τ_g (years)	6×10^5	7×10^6	5×10^7

In figure 1, the evolution of $M_{\rm core}$ and that of $M_{\rm env}$ are shown for the case where $\dot{M}_{\rm core} = 1 \times 10^{-8}$ $M_\oplus$ per year. At the start of this calculation, when $M_{\rm core}$ reached $M_{\rm core}^{\rm crit}$, $M_{\rm env} \simeq 1$ $M_\oplus$. Certainly the gas accretion rate is found to be larger than the core accretion rate, but it takes about 5×10^7 years for $M_{\rm core} \simeq M_{\rm env}$ to be reached. In table 2, the growth times τ_g are listed for three values of $\dot{M}_{\rm core}$. The growth time is defined as the time interval required for $M_{\rm env}$ to increase by a factor of e after $M_{\rm core}$ has become $M_{\rm core}^{\rm crit}$. From this table, τ_g is found to be longer for the smaller values of $\dot{M}_{\rm core}$. This tendency is understood by considering the characteristic time of contraction of the envelope, called the *Kelvin–Helmholtz time*. Roughly speaking, the Kelvin–Helmholtz time is equal to the time interval required for discarding the gravitational energy of the envelope into space from its surface in the form of the luminosity, and is written as $GM_{\rm core}M_{\rm env}/R_{\rm core}L$ when $M_{\rm env} \leqslant M_{\rm core}$, where G is the gravitational constant, $R_{\rm core}$ is the radius of the core and L is the luminosity released owing to the gravitational contraction of the envelope. Just after $M_{\rm core}$ becomes larger than $M_{\rm core}^{\rm crit}$, the luminosity is supplied mainly by the gravitational contraction of the envelope. It is at most as large as that supplied by the falling planetesimals. In table 1 we show the luminosity just when $M_{\rm core}$ reaches $M_{\rm core}^{\rm crit}$ as $L^{\rm crit}$. As seen in table 1, $L^{\rm crit}$ decreases, being nearly proportional to $\dot{M}_{\rm core}$; that is, the Kelvin–Helmholtz time becomes larger as $\dot{M}_{\rm core}$ becomes smaller. That is why it takes a longer time for the gaseous envelope to be attracted from the solar nebula in the case of a smaller $M_{\rm core}^{\rm crit}$.

We conclude that a proto-Jovian core of even just a few Earth masses (not 10 $M_\oplus$) can attract the gaseous envelope from the solar nebula; in this case, however, it takes a long time, of the order of 10^7–10^8 years, for the envelope to become much more massive than 1 $M_\oplus$. From our results, we can deduce that the values of the present core masses of our Jovian planets govern the scenario of the formation of the Jovian planets. If the higher estimates of $M_{\rm core}$ are accurate—more precisely, if $M_{\rm core} \gtrsim 5$ $M_\oplus$—the proto-Jovian core would be able to attract a large amount of the gaseous envelope from the solar nebula within the lifetime of the solar nebula in accordance with the core accretion model. On the other hand, if the lower estimates are accurate, the core accretion model can no longer survive, because the nebular gas would have dissipated before the proto-Jovian core attracted the massive gaseous envelope. In this case, we have to consider another formation scenario, e.g., via the gaseous gravitational instability of the nebular disc [12]. In any case, the value of the core mass of our Jovian planets is the essential information for the formation theory of the Jovian planets. We hope that, as a result of the improvement in the equations of state of hydrogen, silicate and so on under extreme conditions and the interior model of the Jovian planets, we will be able to obtain more accurate values of the core masses of the Jovian planets.

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